$\frac{\partial T}{\partial t} + (\vec{v}\cdot\vec{\nabla})T = D^{(\theta)}\Delta T$ [illegible]

$\vec{\nabla}\cdot\vec{v} = 0$

evolutionary equation for temp. field

(1) $\frac{\partial T}{\partial t} + \vec{\nabla}\cdot(\vec{v}T) = D^{(\theta)}\Delta T$

Mean Field Approach:

$\vec{v} = \vec{\bar{V}} + \vec{u}$, $T = \bar{T} + \theta$ } Reynolds Rules: $\langle \vec{v}\rangle = \vec{\bar{V}} = 0$; $\langle \vec{u}\rangle = 0$; $\langle\theta\rangle = 0$; $\langle T\rangle = \bar{T}$

mean field equation

$\frac{\partial \bar{T}}{\partial t} + \vec{\nabla}\cdot\left[\vec{\bar{V}}\bar{T} + \langle\vec{u}\theta\rangle + \langle\vec{u}\bar{T}\rangle + \langle\vec{\bar{V}}\theta\rangle\right] = D^{(\theta)}\Delta\bar{T}$ (turbulent heat flux)

(2) $\frac{\partial \bar{T}}{\partial t} + \vec{\nabla}\cdot[\langle\vec{u}\theta\rangle] = D^{(\theta)}\Delta\bar{T}$

Equation for temperature fluctuation: (1)-(2)

$\frac{\partial\theta}{\partial t} + \vec{\nabla}\cdot(\vec{u}\theta - \langle\vec{u}\theta\rangle) - D^{(\theta)}\Delta\theta = -\vec{\nabla}\cdot(\vec{u}\bar{T}) = -(\vec{u}\cdot\vec{\nabla})\bar{T}$ (non linear term; $\vec{\nabla}\cdot\vec{u}=0$; source of temp. fluc.; $\vec{\nabla}\bar{T} \times \vec{u}$ tangling)

one way coupling: $\vec{u}\to T$; $T \not\to \vec{u}$

$Pe = \frac{|\vec{\nabla}\cdot(\vec{u}\theta - \langle\vec{u}\theta\rangle)|}{|D^{(\theta)}\Delta\theta|} = \frac{\text{non linear terms}}{\text{diffusion terms}}$

Dimensional Analysis:

$\frac{\theta}{\tau_\theta} = -(\vec{u}\cdot\vec{\nabla})\bar{T}$

$\theta = -\tau_\theta(\vec{u}\cdot\vec{\nabla})\bar{T}$; $\langle u^2\rangle = u_0^2$

$Pe >> 1 \to \tau_\theta = \tau_0 \to \theta = -\tau_0(\vec{u}\cdot\vec{\nabla})\bar{T}$

$\langle u_i\theta\rangle = -\tau_0\langle u_i u_j\rangle\nabla_j\bar{T}$; $\langle u_i u_j\rangle = \frac{\langle u^2\rangle}{3}\delta_{ij}$ (Isotropic turbulence)

$\langle u_i\theta\rangle = -\tau_0\frac{\langle u^2\rangle}{3}\delta_{ij}\nabla_j\bar{T} = -\tau_0\frac{\langle u^2\rangle}{3}\vec{\nabla}\bar{T}$

$\langle\vec{u}\theta\rangle = -D_T\vec{\nabla}\bar{T}$; $D_T = \tau_0\frac{\langle u^2\rangle}{3} = \frac{u_0 l_0}{3}$

$\frac{\partial\bar{T}}{\partial t} + \vec{\nabla}\cdot(-D_T\vec{\nabla}\bar{T}) = D^{(\theta)}\Delta\bar{T}$ [illegible]

$\frac{\partial\bar{T}}{\partial t} - \vec{\nabla}\cdot(D_e\vec{\nabla}\bar{T}) = 0$; $D^{(\theta)} = \text{const}$, $D_e = D^{(\theta)} + D_T$; $\bar{T}_0(\vec{r})$

$\bar{T}(t,\vec{r}) = \frac{1}{(2\pi)^3}\int\bar{T}(t,\vec{k})\exp(i\,\vec{k}\cdot\vec{r})\,d\vec{k}$

$\bar{T}(t,\vec{k}) = \int\bar{T}(t,\vec{r})\exp(-i\vec{k}\cdot\vec{r})\,d\vec{r}$

$\frac{1}{(2\pi)^3}\int\left(\frac{d\bar{T}(t,\vec{k})}{dt} + D_e k^2\bar{T}(t,\vec{k})\right)\exp(i\,\vec{k}\cdot\vec{r})\,d\vec{k} = 0$

$\frac{d\bar{T}(t,\vec{k})}{dt} = -D_e k^2\bar{T}(t,\vec{k})$

$\int\frac{d\bar{T}(t,\vec{k})}{\bar{T}(t,\vec{k})} = -D_e k^2\int dt$

$\bar{T}(t,\vec{k}) = \bar{T}_0(\vec{k})\exp(-D_e k^2 t)$

$\bar{T}_0(\vec{k}) = \int\bar{T}_0(\vec{r}')\exp(-i\,\vec{k}\cdot\vec{r}')\,d\vec{r}'$

$\bar{T}(t,\vec{k}) = \int\bar{T}_0(\vec{r}')\exp(-i\,\vec{k}\cdot\vec{r}' - D_e k^2 t)\,d\vec{r}'$

$\bar{T}(t,\vec{r}) = \frac{1}{(2\pi)^3}\int d\vec{k}\int\bar{T}_0(\vec{r}')\exp\left[i\vec{k}(\vec{r}-\vec{r}') - D_e k^2 t\right]d\vec{r}'$

$i\vec{k}(\vec{r}-\vec{r}') - D_e k^2 t = -D_e t\left(\vec{k} - \frac{i(\vec{r}-\vec{r}')}{2D_e t}\right)^2 - \frac{(\vec{r}-\vec{r}')^2}{4D_e t}$

$I = \frac{1}{(2\pi)^3}\int\exp\left[-D_e t\left(\vec{k} - \frac{i(\vec{r}-\vec{r}')}{2D_e t}\right)^2\right]d\vec{k} = \frac{1}{(2\pi)^3}\int\exp(-\alpha^2\tilde{k}^2)\,d\tilde{k} =$

$= \frac{1}{(2\pi)^3}\int_0^\infty\exp(-\alpha^2\tilde{k}^2)\tilde{k}^2 d\tilde{k}\int_0^\pi\sin\theta\,d\theta\int_0^{2\pi}d\varphi = \frac{1}{(4\pi D_e t)^{3/2}}$; $\alpha = (D_e t)^{1/2}$; $\tilde{k} = \vec{k} - \frac{i(\vec{r}-\vec{r}')}{2D_e t}$

$\bar{T}(t,\vec{r}) = \frac{1}{(4\pi D_e t)^{3/2}}\int\bar{T}_0(\vec{r}')\exp\left[-\frac{(\vec{r}-\vec{r}')^2}{4D_e t}\right]d\vec{r}'$

$Pe << 1 \to \tau_\theta = \frac{l_0^2}{D^{(\theta)}}$

$\theta = -\frac{l_0^2}{D^{(\theta)}}(\vec{u}\cdot\vec{\nabla})\bar{T}$

$\langle u_i\theta\rangle = -\frac{l_0^2}{D^{(\theta)}}\langle u_i u_j\rangle\nabla_j\bar{T}$

$\langle\vec{u}\theta\rangle = -D_T\vec{\nabla}\bar{T}$; $D_T = \frac{u_0 l_0}{3}\frac{u_0 l_0}{D^{(\theta)}}$ (Pe)

$\bar{T}(t,x) = \frac{1}{(4\pi D_e t)^{1/2}}\int\bar{T}_0(x')\exp\left[-\frac{(x-x')^2}{4D_e t}\right]dx'$

$\bar{T}_0(\vec{r}) = T_* \delta(\vec{r})$; $\bar{T}_0(x) = T_*\delta(x)$

$\bar{T}(t,\vec{r}) = \frac{T_*}{(4\pi D_e t)^{3/2}}\exp\left[-\frac{\vec{r}^2}{4D_e t}\right]$

$\bar{T}(t,x) = \frac{T_*}{(4\pi D_e t)^{1/2}}\exp\left[-\frac{x^2}{4D_e t}\right]$

$\bar{T}(t,x=0) \sim T_*/\sqrt{t}$; $L_T \sim \sqrt{D_e t}$

$\tau_{\text{diffusion}} = \frac{L_T^2}{D_e}$

(1) $\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(\vec{v}\,n) = D^{(n)}\Delta n$

Mean field Approach

$\vec{v} = \vec{\bar{V}} + \vec{u}$, $n = \bar{n} + n'$ } Reynolds Rules: $\langle\vec{v}\rangle = \vec{\bar{V}} = 0$; $\langle n\rangle = \bar{n}$; $\langle\vec{u}\rangle = 0$; $\langle n'\rangle = 0$

mean field eq. (turbulent flux of particles)

$\frac{\partial\bar{n}}{\partial t} + \vec{\nabla}\cdot\left[\vec{\bar{V}}\bar{n} + \langle\vec{u}n'\rangle + \langle\vec{u}\bar{n}\rangle + \langle\vec{\bar{V}}n'\rangle\right] = D^{(n)}\Delta\bar{n}$

(2) $\frac{\partial\bar{n}}{\partial t} + \vec{\nabla}\cdot(\langle\vec{u}n'\rangle) = D^{(n)}\Delta\bar{n}$

eq. for particle number density fluc.: (1)-(2)

$\frac{\partial n'}{\partial t} + \vec{\nabla}\cdot(\vec{u}n' - \langle\vec{u}n'\rangle) - D^{(n)}\Delta n' = -(\vec{u}\cdot\vec{\nabla})\bar{n} - \bar{n}(\vec{\nabla}\cdot\vec{u})$ (non linear term; source of particle number density fluc.)

[illegible] eq. for the evolution of instantaneous particle number density

one way coupling $\vec{u}\to n$; $n\not\to\vec{u}$; non inertial particles

$Pe = \frac{|\vec{\nabla}\cdot(\vec{u}n' - \langle\vec{u}n'\rangle)|}{|D^{(n)}\Delta n'|} = \frac{\text{non linear terms}}{\text{diffusion terms}}$

Dimensional Analysis:

$\frac{n'}{t_n} = -(\vec{u}\cdot\vec{\nabla})\bar{n} - \bar{n}(\vec{\nabla}\cdot\vec{u})$

$n' = -t_n\left[(\vec{u}\cdot\vec{\nabla})\bar{n} + \bar{n}(\vec{\nabla}\cdot\vec{u})\right]$

$Pe >> 1 \to t_n = \tau_0 \to n' = -\tau_0\left[(\vec{u}\cdot\vec{\nabla})\bar{n} + \bar{n}(\vec{\nabla}\cdot\vec{u})\right]$

$\langle u_i n'\rangle = -\tau_0\left[\langle u_i u_j\rangle\nabla_j\bar{n} - \bar{n}\langle u_i u_j\rangle\frac{\nabla_j\bar{\rho}}{\bar{\rho}}\right]$

$\langle u_i u_j\rangle = \frac{\langle u^2\rangle}{3}\delta_{ij}$ (isotropic turbulence)

$\langle u_i n'\rangle = -\tau_0\frac{\langle u^2\rangle}{3}\delta_{ij}\left[\nabla_j\bar{n} + \lambda_j\bar{n}\right]$; $\vec{\lambda} = -\frac{\vec{\nabla}\bar{\rho}}{\bar{\rho}}$

$\langle u_i n'\rangle = -\frac{\tau_0\langle u^2\rangle}{3}\left[\vec{\nabla}\bar{n} + \vec{\lambda}\bar{n}\right]$

$\langle\vec{u}n'\rangle = \vec{V}^{eff}\bar{n} - D_T\vec{\nabla}\bar{n}$; $\vec{V}^{eff} = D_T\frac{\vec{\nabla}\bar{\rho}}{\bar{\rho}} = D_T\left(\frac{\vec{\nabla}\bar{P}}{\bar{P}} - \frac{\vec{\nabla}\bar{T}}{\bar{T}}\right)$

$Pe << 1 \to t_n = \frac{l_0^2}{D^{(n)}}$; $\langle u^2\rangle = u_0^2$; $D_T = \frac{\tau_0\langle u^2\rangle}{3} = \frac{u_0 l_0}{3}$

$n' = -\frac{l_0^2}{D^{(n)}}\left[(\vec{u}\cdot\vec{\nabla})\bar{n} + \bar{n}(\vec{\nabla}\cdot\vec{u})\right]$

$\langle u_i n'\rangle = -\frac{l_0^2}{D^{(n)}}\left[\langle u_i u_j\rangle\nabla_j\bar{n} - \bar{n}\langle u_i u_j\rangle\frac{\nabla_j\bar{\rho}}{\bar{\rho}}\right]$

$\langle u_i u_j\rangle = \frac{\langle u^2\rangle}{3}\delta_{ij}$ (isotropic turbulence)

$\langle u_i n'\rangle = -\frac{l_0^2}{D^{(n)}}\frac{\langle u^2\rangle}{3}\delta_{ij}\left[\nabla_j\bar{n} + \lambda_j\bar{n}\right]$; $\vec{\lambda} = -\frac{\vec{\nabla}\bar{\rho}}{\bar{\rho}}$

$\langle\vec{u}n'\rangle = \vec{V}^{eff}\bar{n} - D_T\vec{\nabla}\bar{n}$; $\vec{V}^{eff} = D_T\frac{\vec{\nabla}\bar{P}}{\bar{P}} = D_T\left(\frac{\vec{\nabla}\bar{P}}{\bar{P}} - \frac{\vec{\nabla}\bar{T}}{\bar{T}}\right)$; $D_T = \frac{u_0 l_0}{3}Pe$

$\vec{\nabla}\cdot(\rho\vec{u}) = 0$; $\frac{\partial\rho}{\partial t} \propto Ma^2 << 1$ (anelastic approximation)

$\rho(\vec{\nabla}\cdot\vec{u}) + (\vec{u}\cdot\vec{\nabla})\rho = 0$

$\vec{\nabla}\cdot\vec{u} = -\frac{(\vec{u}\cdot\vec{\nabla})\rho}{\rho} \approx -\frac{(\vec{u}\cdot\vec{\nabla})\bar{\rho}}{\bar{\rho}}$

[illegible]

$\frac{\partial\bar{n}}{\partial t} + \vec{\nabla}\cdot\left(\vec{V}^{eff}\bar{n} - D_T\vec{\nabla}\bar{n}\right) = D^{(n)}\Delta\bar{n}$

$\frac{\partial\bar{n}}{\partial t} + \vec{\nabla}\cdot\left(\vec{V}^{eff}\bar{n} - (D_T + D^{(n)})\vec{\nabla}\bar{n}\right) = 0$

[illegible]

$\vec{\nabla}\cdot\left(\vec{V}^{eff}\bar{n} - (D_T + D^{(n)})\vec{\nabla}\bar{n}\right) = 0$

$\vec{V}^{eff}\bar{n} = (D_T + D^{(n)})\vec{\nabla}\bar{n} + \text{const}$ [illegible]

$D_T\frac{\vec{\nabla}\bar{P}}{\bar{P}} = (D_T + D^{(n)})\frac{\vec{\nabla}\bar{n}}{\bar{n}}$

$-D_T\frac{\vec{\nabla}\bar{T}}{\bar{T}} = (D_T + D^{(n)})\frac{\vec{\nabla}\bar{n}}{\bar{n}}$

$-\frac{D_T}{D_T + D^{(n)}}\ln\left(\frac{\bar{T}}{T_0}\right) = \ln\left(\frac{\bar{n}}{n_0}\right)$

$\frac{\bar{n}}{n_0} = \left(\frac{\bar{T}}{T_0}\right)^{-\frac{D_T}{D_T + D^{(n)}}}$; $\bar{T}(z = z_0) = T_0$; $\bar{n}(z = z_0) = n_0$

$\bar{T} = \bar{T}_0\exp\left[-\frac{(z - z_0)^2}{L_T^2}\right] \Rightarrow \frac{\bar{n}}{n_0} = \exp\left[\frac{D_T(z - z_0)^2}{L_T^2(D_T + D^{(n)})}\right] = \exp\left[\frac{(z - z_0)^2}{L_n^2}\right]$

$L_n = \sqrt{\frac{D_T + D^{(n)}}{D_T}}\,L_T \xrightarrow{Pe << 1} \sqrt{\frac{D^{(n)}}{D_T}}\,L_T$

$\xrightarrow{Pe >> 1} = L_T$

evolutionary eq. for temp. field

$\frac{\partial T}{\partial t} + (\vec{v}\cdot\vec{\nabla})T = D^{(\theta)}\Delta T$; $\vec{\nabla}\cdot\vec{v} = 0$

(1) $\frac{\partial T}{\partial t} + \vec{\nabla}\cdot(\vec{v}T) = D^{(\theta)}\Delta T$ [illegible]

mean field approach:

$\vec{v} = \vec{\bar{V}} + \vec{u}$, $T = \bar{T} + \theta$ } Reynolds Rules: $\langle\vec{v}\rangle = \vec{\bar{V}} = 0$; $\langle T\rangle = \bar{T}$; $\langle\vec{u}\rangle = 0$; $\langle\theta\rangle = 0$

Mean field eq.:

$\frac{\partial\bar{T}}{\partial t} + \vec{\nabla}\cdot\left[\vec{\bar{V}}\bar{T} + \langle\vec{u}\theta\rangle + \langle\vec{u}\bar{T}\rangle + \langle\vec{\bar{V}}\theta\rangle\right] = D^{(\theta)}\Delta\bar{T}$ (turbulent heat flux)

(2) $\frac{\partial\bar{T}}{\partial t} + \vec{\nabla}\cdot(\langle\vec{u}\theta\rangle) = D^{(\theta)}\Delta\bar{T}$

Eq. for temp. fluc.: (1)-(2)

$\frac{\partial\theta}{\partial t} + \vec{\nabla}\cdot(\vec{u}\theta - \langle\vec{u}\theta\rangle) - D^{(\theta)}\Delta\theta = -(\vec{\nabla}\cdot\vec{u})\bar{T} = -(\vec{u}\cdot\vec{\nabla})\bar{T}$ (non linear term; $Pe << 1$; $\vec{\nabla}\cdot\vec{u} = 0$; source of temp fluc.; $\vec{\nabla}\bar{T}$ tangling)

one way coupling: $\vec{u}\to T$; $T\not\to\vec{u}$

$Pe = \frac{|\vec{\nabla}\cdot(\vec{u}\theta - \langle\vec{u}\theta\rangle)|}{|D^{(\theta)}\Delta\theta|} = \frac{\text{non linear terms}}{\text{diffusion terms}}$

$\theta(t,\vec{x}) = \iint\theta(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$u_i(t,\vec{x}) = \iint u_i(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$\frac{\partial\theta(t,\vec{x})}{\partial t} = \iint(i\omega)\theta(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$\Delta\theta(t,\vec{x}) = \iint(ik)^2\theta(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$\iint\left[(D^{(\theta)}k^2 + i\omega)\theta(\omega,\vec{k}) + u_j(\omega,\vec{k})\nabla_j\bar{T}\right]\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k} = 0$

$\theta(\omega,\vec{k}) = -\frac{u_j(\omega,\vec{k})\nabla_j\bar{T}}{D^{(\theta)}k^2 + i\omega}$

multi scale approach:

$\langle u_i(t,\vec{x})\theta(t,\vec{x})\rangle = \lim_{\substack{\vec{x}\to\vec{y}\\ t_1\to t_2}}\langle u_i(t_1,\vec{x})\theta(t_2,\vec{y})\rangle = \iint\langle u_i(\omega,\vec{k})\theta(-\omega,-\vec{k})\rangle\,d\omega\,d\vec{k}$

$\langle u_i(t,\vec{x})\theta(t,\vec{x})\rangle = -\nabla_j\bar{T}\iint\frac{\langle u_i(\omega,\vec{k})u_j(-\omega,-\vec{k})\rangle}{D^{(\theta)}k^2 - i\omega}\,d\omega\,d\vec{k}$

model of isotropic, homogeneous and incompressible turbulence:

$\langle u_i(\omega,\vec{k})u_j(-\omega,-\vec{k})\rangle = \frac{\langle u^2\rangle E(k)\phi(\omega)}{8\pi k^2}\left[\delta_{ij} - \frac{k_i k_j}{k^2}\right]$; $\langle u^2\rangle = u_0^2$

$\langle u\theta\rangle = -\frac{\langle u^2\rangle}{8\pi}\nabla_j\bar{T}\int_{k_0}^{k_\nu}\frac{E(k)k^2}{k^2}dk\int_{-\infty}^{\infty}\frac{\phi(\omega)}{D^{(\theta)}k^2 - i\omega}d\omega\int_0^{2\pi}d\varphi\int_0^\pi\sin\theta\,d\theta\left[\delta_{ij} - \frac{k_i k_j}{k^2}\right]$

$\langle u\theta\rangle = -D_T\vec{\nabla}\bar{T}$; $D_T = \frac{q-1}{q+1}\frac{u_0 l_0}{3}Pe$; for kolmogorov $q = \frac{5}{3}$: $D_T = \frac{u_0 l_0}{12}Pe$

$\vec{\nabla}\cdot\vec{u} = 0$

$n'(t,\vec{x}) = \iint n'(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$\frac{\partial n'(t,\vec{x})}{\partial t} = \iint(i\omega)n'(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$ [illegible]

$\Delta n'(t,\vec{x}) = \iint(ik)^2 n'(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$u_i(t,\vec{x}) = \iint u_i(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$\nabla_j u_i(t,\vec{x}) = \iint(i\,k_j)u_i(\omega,\vec{k})\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k}$

$\iint\left[(D^{(n)}k^2 + i\omega)n'(\omega,\vec{k}) + (\nabla_j\bar{n} + i\,k_j\bar{n})u_j(\omega,\vec{k})\right]\exp(i\,\vec{k}\cdot\vec{x} + i\omega t)\,d\omega\,d\vec{k} = 0$

$n'(\omega,\vec{k}) = -\frac{(\nabla_j\bar{n} + i\,k_j\bar{n})u_j(\omega,\vec{k})}{D^{(n)}k^2 + i\omega}$ multi scale approach;

$\langle u_i(t,\vec{x})n'(t,\vec{x})\rangle = \lim_{\substack{\vec{x}\to\vec{y}\\ t_1\to t_2}}\langle u_i(t_1,\vec{x})n'(t_2,\vec{y})\rangle = \iint\langle u_i(\omega,\vec{k})n'(-\omega,-\vec{k})\rangle\,d\omega\,d\vec{k}$

$\langle u_i(t,\vec{x})n'(t,\vec{x})\rangle = -\iint\frac{(\nabla_j\bar{n} - i\,k_j\bar{n})}{D^{(n)}k^2 - i\omega}\langle u_i(\omega,\vec{k})u_j(-\omega,-\vec{k})\rangle\,d\omega\,d\vec{k}$

model of isotropic homogeneous turbulence in anelastic approximation:

$\langle u_i(\omega,\vec{k})u_j(-\omega,-\vec{k})\rangle = \frac{\langle u^2\rangle E(k)\phi(\omega)}{8\pi k^2}\left[\delta_{ij} - \frac{k_i k_j}{k^2} + \frac{i}{k^2}(\lambda_i k_j - \lambda_j k_i)\right]$

$\langle\vec{u}n'\rangle = -\frac{\langle u^2\rangle}{8\pi}\int\frac{E(k)k^2}{k^2}dk\int\frac{\phi(\omega)}{D^{(n)}k^2 - i\omega}d\omega\int_0^{2\pi}d\varphi\int_0^\pi\sin\theta\,d\theta\left(\nabla_j\bar{n} - i\,k_j\bar{n}\right)\left[\delta_{ij} - \frac{k_i k_j}{k^2} + \frac{i}{k^2}(\lambda_i k_j - \lambda_j k_i)\right]$

$\langle\vec{u}n'\rangle = -D_T\vec{\nabla}\bar{n} + \vec{V}^{eff}\bar{n}$; $\vec{V}^{eff} = -D_T\vec{\lambda} = D_T\frac{\vec{\nabla}\bar{\rho}}{\bar{\rho}} = D_T\left(\frac{\vec{\nabla}\bar{P}}{\bar{P}} - \frac{\vec{\nabla}\bar{T}}{\bar{T}}\right)$; $\langle u^2\rangle = u_0^2$

$D_T = \frac{q-1}{q+1}\frac{u_0 l_0}{3}Pe$; for kolmogorov $q = \frac{5}{3}$ $D_T = \frac{u_0 l_0}{12}Pe$

evolutionary eq. for temp. field:

$$\frac{\partial T}{\partial t} + (\vec{v}\cdot\vec{\nabla})T = D^{(\theta)}\Delta T$$

$\vec{\nabla}\cdot\vec{u} = 0$ (T approach SdC)

(1) $\frac{\partial T}{\partial t} + \vec{\nabla}(\vec{v}T) = D^{(\theta)}\Delta T$

Mean Field Approach:

$\vec{v} = \overline{\vec{v}} + \vec{u}$, $T = \overline{T} + \theta$ Reynolds Rules: $\langle \vec{v}\rangle = \overline{\vec{v}} = 0$; $\langle T\rangle = \overline{T}$; $\langle \vec{u}\rangle = 0$; $\langle\theta\rangle = 0$

mean field eq. — turbulent heat flux

$$\frac{\partial \overline{T}}{\partial t} + \vec{\nabla}\cdot\left[\overline{\vec{v}}\,\overline{T} + \langle\vec{u}\theta\rangle + \langle \vec{u}\rangle\overline{T} + \langle\overline{\vec{v}}\theta\rangle\right] = D^{(\theta)}\Delta\overline{T}$$

(2) $\frac{\partial \overline{T}}{\partial t} + \vec{\nabla}\cdot\langle\vec{u}\theta\rangle = D^{(\theta)}\Delta\overline{T}$

eq. for temp. fluc.: (1)−(2) $\quad \vec{\nabla}\cdot\vec{u} = 0$

$$\frac{\partial\theta}{\partial t} + \underbrace{\vec{\nabla}\cdot(\vec{u}\theta - \langle\vec{u}\theta\rangle)}_{\text{non linear term}} - D^{\theta}\Delta\theta = -\vec{\nabla}\cdot(\vec{u}\,\overline{T}) = \underbrace{-(\vec{u}\cdot\vec{\nabla})\overline{T}}_{\text{source of temp. fluc.}}$$

$\nabla\overline{T}$ — $\vec{u}$ tangling

one way coupling: $\vec{u}\to T$; $T \not\to \vec{u}$

$$Pe = \frac{|\vec{\nabla}\cdot(\vec{u}\theta - \langle\vec{u}\theta\rangle)|}{|D^{\theta}\Delta\theta|} = \frac{\text{non linear terms}}{\text{diffusion terms}} ;\quad Re = \frac{|(\vec{u}\cdot\vec{\nabla})\vec{u}|}{|\nu\Delta\vec{u}|} = \frac{\text{non linear term}}{\text{viscous term}}$$

$Pe \gg 1$; $Re \gg 1$

$$\frac{\partial\theta(t,-\vec{k})}{\partial t} = -u_j(t,-\vec{k})\nabla_j\overline{T} - Q^N(t,-\vec{k}) ;\quad Q^N = \vec{\nabla}\cdot(\vec{u}\theta - \langle\vec{u}\theta\rangle) - D^{(\theta)}\Delta\theta$$

$$\frac{\partial u_i(t,\vec{k})}{\partial t} = u_i^N(t,\vec{k}) ;\quad \vec{u}^N = -(\vec{u}\cdot\vec{\nabla})\vec{u} - \frac{\vec{\nabla}p}{\rho} + \nu\Delta\vec{u}$$

$F_i(t,\vec{k}) = \langle u_i(t,\vec{k})\theta(t,-\vec{k})\rangle$, $F_{ij}(t,\vec{k}) = \langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle$ — second order moments

① $\frac{\partial F_i(t,\vec{k})}{\partial t} = \left\langle\frac{\partial u_i(t,\vec{k})}{\partial t}\theta(t,-\vec{k})\right\rangle + \left\langle u_i(t,\vec{k})\frac{\partial\theta(t,-\vec{k})}{\partial t}\right\rangle = -\underbrace{\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle}_{F_{ij}(\vec{k})}\nabla_j\overline{T} + \hat{M}F_i(t,\vec{k})$

$\hat{M}F_i(t,\vec{k}) = \langle u_i^N(t,\vec{k})\theta(t,-\vec{k})\rangle - \langle u_i(t,\vec{k})Q^N(t,-\vec{k})\rangle$ — third moments

② $\frac{\partial F_{ij}(t,\vec{k})}{\partial t} = \left\langle\frac{\partial u_i(t,\vec{k})}{\partial t}u_j(t,-\vec{k})\right\rangle + \left\langle u_i(t,\vec{k})\frac{\partial u_j(t,-\vec{k})}{\partial t}\right\rangle = \hat{M}F_{ij}(t,\vec{k})$

$\hat{M}F_{ij}(t,\vec{k}) = \langle u_i^N(t,\vec{k})u_j(t,-\vec{k})\rangle + \langle u_i(t,\vec{k})u_j^N(t,-\vec{k})\rangle$

Basic state: $\vec{\nabla}\overline{T} = 0$

③ $\frac{\partial F_i^{(0)}(t,\vec{k})}{\partial t} = \hat{M}F_i^{(0)}(t,\vec{k})$

④ $\frac{\partial F_{ij}^{(0)}(t,\vec{k})}{\partial t} = \hat{M}F_{ij}^{(0)}(t,\vec{k})$

①−③ ⇒ $\frac{\partial F_i(t,\vec{k})}{\partial t} - \frac{\partial F_i^{(0)}(t,\vec{k})}{\partial t} = -\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle\nabla_j\overline{T} + \hat{M}F_i(t,\vec{k}) - \hat{M}F_i^{(0)}(t,\vec{k})$, with $\hat{M}F_i - \hat{M}F_i^{(0)} = -\frac{1}{\tau(k)}\left[F_i(t,\vec{k}) - F_i^{(0)}(t,\vec{k})\right]$

②−④ ⇒ $\frac{\partial F_{ij}(t,\vec{k})}{\partial t} - \frac{\partial F_{ij}^{(0)}(t,\vec{k})}{\partial t} = \hat{M}F_{ij}(t,\vec{k}) - \hat{M}F_{ij}^{(0)}(t,\vec{k}) = -\frac{1}{\tau(k)}\left[F_{ij}(t,\vec{k}) - F_{ij}^{(0)}(t,\vec{k})\right]$

$\tau(k) \ll t$ ⇒ steady state $\frac{\partial}{\partial t} = 0$

$F_i(t,\vec{k}) = F_i^{(0)}(t,\vec{k}) - \tau(k)\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle\nabla_j\overline{T}$ (no flux at basic state)

$F_{ij}(t,\vec{k}) = F_{ij}^{(0)}(t,\vec{k})$

$$F_i(t,\vec{x}) = \langle u_i(t,\vec{x})\theta(t,\vec{x})\rangle = \lim_{\vec{x}\to\vec{y}}\langle u_i(t,\vec{x})\theta(t,\vec{y})\rangle = \int\langle u_i(t,\vec{k})\theta(t,-\vec{k})\rangle d\vec{k} \equiv \int F_i(t,\vec{k})d\vec{k}$$

$$F_i(t,\vec{x}) = -\int\tau(k)\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle^{(0)}d\vec{k}\,(\nabla_j\overline{T})$$

model of isotropic, homogeneous and incompressible turbulence:

$$\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle^{(0)} = \frac{\langle u^2\rangle E(k)}{8\pi k^2}\left[\delta_{ij} - \frac{k_ik_j}{k^2}\right];\quad \langle u^2\rangle = u_0^2$$

$$\langle\vec{u}\theta\rangle = -\frac{\langle u^2\rangle}{8\pi}\nabla_j\overline{T}\int_{k_0}^{k_\nu}\frac{\tau(k)E(k)k^2}{k^2}dk\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta\left[\delta_{ij} - \frac{k_ik_j}{k^2}\right]$$

$\langle\vec{u}\theta\rangle = -D_T\vec{\nabla}\overline{T}$; $D_T = \tau_0\frac{\langle u^2\rangle}{3} = \frac{u_0 l_0}{3}$

eq. for the evolution of instantaneous particle number density

(1) $\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(\vec{v}n) = D^{(n)}\Delta n$

mean field approach:

$\vec{v} = \overline{\vec{v}} + \vec{u}$, $n = \overline{n} + n'$ Reynolds Rules: $\langle\vec{v}\rangle = \overline{\vec{v}} = 0$; $\langle n\rangle = \overline{n}$; $\langle\vec{u}\rangle = 0$; $\langle n'\rangle = 0$

mean field eq.: turbulent flux of particles

$\frac{\partial\overline{n}}{\partial t} + \vec{\nabla}\cdot\left[\overline{\vec{v}}\,\overline{n} + \langle\vec{u}n'\rangle + \langle\vec{u}\rangle\overline{n} + \langle\overline{\vec{v}}n'\rangle\right] = D^{(n)}\Delta\overline{n}$

(2) $\frac{\partial\overline{n}}{\partial t} + \vec{\nabla}\cdot\langle\vec{u}n'\rangle = D^{(n)}\Delta\overline{n}$

$\vec{\nabla}(\rho\vec{u}) = 0$, $\frac{\partial\rho}{\partial t} \propto Ma^2 \ll 1$ anelastic approximation

$\rho\,\vec{\nabla}\cdot\vec{u} + (\vec{u}\cdot\vec{\nabla})\rho = 0$

$\vec{\nabla}\cdot\vec{u} = -\frac{(\vec{u}\cdot\vec{\nabla})\rho}{\rho} \approx -\frac{(\vec{u}\cdot\vec{\nabla})\overline{\rho}}{\overline{\rho}}$ (T approach $\vec{\nabla}\cdot\vec{u}\neq 0$)

eq. for particle number density fluc.: (1)−(2)

$$\frac{\partial n'}{\partial t} + \underbrace{\vec{\nabla}\cdot(\vec{u}n' - \langle\vec{u}n'\rangle)}_{\text{non linear term}} - D^{(n)}\Delta n' = \underbrace{-(\vec{u}\cdot\vec{\nabla})\overline{n} - \overline{n}(\vec{\nabla}\cdot\vec{u})}_{\text{source of particle number density fluc.}}$$

one way coupling: $\vec{u}\to n$; $n \not\to \vec{u}$

non inertial particles

$$Pe = \frac{|\vec{\nabla}\cdot(\vec{u}n' - \langle\vec{u}n'\rangle)|}{|D^{(n)}\Delta n'|} = \frac{\text{non linear terms}}{\text{diffusion term}} ;\quad Re = \frac{|(\vec{u}\cdot\vec{\nabla})\vec{u}|}{|\nu\Delta\vec{u}|} = \frac{\text{non linear term}}{\text{viscous term}}$$

$Pe \gg 1$; $Re \gg 1$

$$\frac{\partial n'(t,-\vec{k})}{\partial t} = -(\vec{u}\cdot\vec{\nabla})\overline{n} - \overline{n}(\vec{\nabla}\cdot\vec{u}) - Q^N(t,-\vec{k}) ;\quad Q^N = \vec{\nabla}\cdot(\vec{u}n' - \langle\vec{u}n'\rangle) - D^{(n)}\Delta n'$$

$$\frac{\partial u_i(t,\vec{k})}{\partial t} = u_i^N(t,\vec{k}) ;\quad \vec{u}^N = -(\vec{u}\cdot\vec{\nabla})\vec{u} - \frac{\vec{\nabla}p}{\rho} + \nu\Delta\vec{u}$$

$F_i(t,\vec{k}) = \langle u_i(t,\vec{k})n'(t,-\vec{k})\rangle$, $F_{ij}(t,\vec{k}) = \langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle$ — second order moments

① $\frac{\partial F_i(t,\vec{k})}{\partial t} = \left\langle\frac{\partial u_i(t,\vec{k})}{\partial t}n'(t,-\vec{k})\right\rangle + \left\langle u_i(t,\vec{k})\frac{\partial n'(t,-\vec{k})}{\partial t}\right\rangle = -\underbrace{\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle}_{F_{ij}(\vec{k})}\nabla_j\overline{n} + ⊕$ (source)

$\hat{M}F_i(t,\vec{k}) = \langle u_i^N(t,\vec{k})n'(t,-\vec{k})\rangle - \langle u_i(t,\vec{k})Q^N(t,-\vec{k})\rangle$ ⊕ ; $ik_j\overline{n}\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle + \hat{M}F_i(t,\vec{k})$ — third moments

② $\frac{\partial F_{ij}(t,\vec{k})}{\partial t} = \left\langle\frac{\partial u_i(t,\vec{k})}{\partial t}u_j(t,-\vec{k})\right\rangle + \left\langle u_i(t,\vec{k})\frac{\partial u_j(t,-\vec{k})}{\partial t}\right\rangle = \hat{M}F_{ij}(t,\vec{k})$

$\hat{M}F_{ij}(t,\vec{k}) = \langle u_i^N(t,\vec{k})u_j(t,-\vec{k})\rangle + \langle u_i(t,\vec{k})u_j^N(t,-\vec{k})\rangle$

Basic state: $\vec{\nabla}\overline{n} = 0$, $\vec{\nabla}\cdot\vec{u} = 0$

③ $\frac{\partial F_i^{(0)}(t,\vec{k})}{\partial t} = \hat{M}F_i^{(0)}(t,\vec{k})$; ④ $\frac{\partial F_{ij}^{(0)}(t,\vec{k})}{\partial t} = \hat{M}F_{ij}^{(0)}(t,\vec{k})$

①−③ ⇒ $\frac{\partial F_i(t,\vec{k})}{\partial t} - \frac{\partial F_i^{(0)}(t,\vec{k})}{\partial t} = (ik_j\overline{n} - \nabla_j\overline{n})\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle + \hat{M}F_i(t,\vec{k}) - \hat{M}F_i^{(0)}(t,\vec{k})$, with $\hat{M}F_i - \hat{M}F_i^{(0)} = -\frac{1}{\tau(k)}\left[F_i(t,\vec{k}) - F_i^{(0)}(t,\vec{k})\right]$

②−④ ⇒ $\frac{\partial F_{ij}(t,\vec{k})}{\partial t} - \frac{\partial F_{ij}^{(0)}(t,\vec{k})}{\partial t} = \hat{M}F_{ij}(t,\vec{k}) - \hat{M}F_{ij}^{(0)}(t,\vec{k}) = -\frac{1}{\tau(k)}\left[F_{ij}(t,\vec{k}) - F_{ij}^{(0)}(t,\vec{k})\right]$

$\tau(k) \ll t$ ⇒ steady state $\frac{\partial}{\partial t} = 0$

$F_i(t,\vec{k}) = F_i^{(0)}(\vec{k}) + \tau(k)(ik_j\overline{n} - \nabla_j\overline{n})\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle$ (no flux at basic state)

$F_{ij}(t,\vec{k}) = F_{ij}^{(0)}(t,\vec{k})$

$$F_i(t,\vec{x}) = \langle u_i(t,\vec{x})n'(t,\vec{x})\rangle = \lim_{\vec{x}\to\vec{y}}\langle u_i(t,\vec{x})n'(t,\vec{y})\rangle = \int\langle u_i(t,\vec{k})n'(t,-\vec{k})\rangle d\vec{k} \equiv \int F_i(t,\vec{k})d\vec{k}$$

$$F_i(t,\vec{x}) = \int\tau(k)(ik_j\overline{n} - \nabla_j\overline{n})\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle^{(0)}d\vec{k}$$

model of isotropic, homogeneous turbulence in anelastic approximation:

$$\langle u_i(t,\vec{k})u_j(t,-\vec{k})\rangle^{(0)} = \frac{\langle u^2\rangle E(k)}{8\pi k^2}\left[\delta_{ij} - \frac{k_ik_j}{k^2} + \frac{i}{k^2}(\lambda_ik_j - \lambda_jk_i)\right]$$

$\langle u^2\rangle = u_0^2$

$$\langle\vec{u}n'\rangle = \frac{\langle u^2\rangle}{8\pi}\int_{k_0}^{k_\nu}\frac{\tau(k)E(k)k^2}{k^2}dk\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta\,(ik_j\overline{n} - \nabla_j\overline{n})\left[\delta_{ij} - \frac{k_ik_j}{k^2} + \frac{i}{k^2}(\lambda_ik_j - \lambda_jk_i)\right]$$

$\langle\vec{u}n'\rangle = \vec{V}^{eff}\overline{n} - D_T\vec{\nabla}\overline{n}$; $\vec{V}^{eff} = -D_T\vec{\lambda} = D_T\frac{\vec{\nabla}\overline{\rho}}{\overline{\rho}} = D_T\left(\frac{\vec{\nabla}\overline{P}}{\overline{P}} - \frac{\vec{\nabla}\overline{T}}{\overline{T}}\right)$; $D_T = \frac{\tau_0\langle u^2\rangle}{3} = \frac{u_0l_0}{3}$

$E(k) = (q-1)k_0^{-1}\left(\frac{k}{k_0}\right)^{-q}$; $\phi(\omega) = \left[\pi\tau_c(\omega^2 + \tau_c^{-2})\right]^{-1}$

$\tau(k) = 2\tau_0\left(\frac{k}{k_0}\right)^{1-q}$; $\tau_c \gg [Dk^2]^{-1}$

$\int dk\int\sin\theta d\theta\,\delta_{ij} = 4\pi\delta_{ij}$; $d\vec{k} = k^2\sin\theta d\theta d\varphi dk$; $k_0 = \frac{1}{l_0}$; $\int\phi(\omega)d\omega = 1$

$\int d\varphi\int\sin\theta d\theta\frac{k_ik_j}{k^2} = \frac{4\pi}{3}\delta_{ij}$; $\int\int\left(\frac{k_i}{k}\right)\sin\theta d\theta d\varphi = 0$

$$\int_{-\infty}^{\infty}\frac{\phi(\omega)}{i\omega + Dk^2} = \frac{1}{\pi\tau_c[\tau_c^{-1} + Dk^2]} \approx \frac{1}{Dk^2}$$

$\int_{k_0}^{k_\nu}E(k)dk = 1$; $\int_{k_0}^{k_\nu}\frac{E(k)}{k^2}dk = \left(\frac{q-1}{q+1}\right)l_0^2$

$\int_{k_0}^{k_\nu}\tau(k)E(k)dk = \tau_0$; $Re \gg 1$; $q = \frac{5}{3}$

$D^{(\theta)} = (u_t$ thermal vel. $)\cdot($ mean free path $)$, $u_t = \sqrt{\frac{k_BT}{m_p}}$; $D^{(n)} = \frac{k_BT}{6\pi\rho\nu a_p}$ ($a_p \ll l_\nu$)

$\varepsilon = \frac{u^2}{\tau} = \frac{u_0^3}{l_0} = \frac{u^3}{l} = \frac{u_\nu^3}{l_\nu}$

$u_0 = (\varepsilon l_0)^{1/3}$; $\tau_0 = \frac{l_0}{u_0} = \varepsilon^{-1/3}l_0^{2/3}$

$Re = \frac{\varepsilon^{1/3}l_0^{4/3}}{\nu}$; $\frac{Re_l}{Re} = \left(\frac{l}{l_0}\right)^{4/3}$

$l_\nu = \frac{l_0}{Re^{3/4}}$; $u_\nu = \frac{u_0}{Re^{1/4}}$; $\tau_\nu = \frac{\tau_0}{Re^{1/2}}$

eq. of motion for inertial particles: $\vec{V}_p \neq \vec{u}$

$$\frac{d\vec{V}_p}{dt} = -\frac{\vec{V}_p - \vec{V}}{\tau_p} + \vec{g} ;\quad \overline{\vec{V}} = 0 \Rightarrow \vec{V}_p = \vec{u} - \tau_p\frac{d\vec{V}_p}{dt} + \tau_p\vec{g}$$

($\tau_p \ll \tau_\nu$, $Re \gg 1$)

$\overline{\vec{V}}_p + \vec{V}_p' = \vec{u} - \tau_p\left(\frac{d\overline{\vec{V}}_p}{dt} + \frac{d\vec{V}_p'}{dt}\right) + \tau_p\vec{g}$; $\overline{\vec{V}}_p = \tau_p\vec{g}$, $\vec{V}_p' = \vec{u} - \tau_p\frac{d\vec{V}_p'}{dt}$

$\vec{V}_p' = \vec{V}_p'^{(0)} + \vec{V}_p'^{(1)} + \dots$; $\vec{V}_p'^{(0)} = \vec{u}$, $\vec{V}_p'^{(1)} = -\tau_p\frac{d\vec{V}_p'^{(0)}}{dt} = -\tau_p\frac{d\vec{u}}{dt}$ ⇒ $\vec{V}_p' = \vec{u} - \tau_p\frac{d\vec{u}}{dt} + o(\tau_p^2)$

$$\vec{V}_p' = \vec{u} - \tau_p\left(\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right) + o(\tau_p^2)$$

$\vec{V}_p = \overline{\vec{V}}_p + \vec{V}_p' = \vec{u} - \tau_p\left(\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right) + \tau_p\vec{g} + o(\tau_p^2)$

$\langle\vec{V}_p\rangle \equiv \overline{\vec{V}}_p = \langle\vec{u}\rangle - \tau_p\left(\frac{\partial}{\partial t}\langle\vec{u}\rangle + \langle(\vec{u}\cdot\vec{\nabla})\vec{u}\rangle + \tau_p\vec{g}\right) + o(\tau_p^2)$ — turbophoresis

$\langle(\vec{u}\cdot\vec{\nabla})\vec{u}\rangle = \langle u_j\nabla_ju_i\rangle = \langle\nabla_j(u_iu_j)\rangle - \langle u_i(\nabla_ju_j)\rangle = \nabla_j\delta_{ij}\frac{\langle u^2\rangle}{3} = \vec{\nabla}\frac{\langle u^2\rangle}{3}$

$$\overline{\vec{V}}_p = -\frac{\tau_p}{3}\vec{\nabla}\langle u^2\rangle + \tau_p\vec{g}$$

$\frac{\partial\overline{n}}{\partial t} + \vec{\nabla}\cdot(\overline{n}\,\overline{\vec{V}}_p + \langle n'\vec{V}_p'\rangle) = D^{(n)}\Delta\overline{n}$; $\langle n'\vec{V}_p'\rangle = \vec{V}^{eff}\overline{n} - D_T\vec{\nabla}\overline{n}$

$\vec{V}_p = -\tau\langle\vec{u}(\vec{\nabla}\cdot\vec{u})\rangle - \tau\langle\vec{V}_p'(\vec{\nabla}\cdot\vec{V}_p')\rangle$; $D_T = \frac{\tau_0\langle\vec{V}_p'^2\rangle}{3}$; $\tau_0 \gg 1$

$\vec{\nabla}\cdot\vec{V}_p' = \vec{\nabla}\cdot\vec{u} - \tau_p\left(\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{u}) + \vec{\nabla}((\vec{u}\cdot\vec{\nabla})\vec{u}) + \vec{\nabla}(\tau_p\vec{g})\right) + o(\tau_p^2)$; $D_T = \frac{\tau_0\langle\vec{V}_p'^2\rangle}{3}$; $\frac{q-1}{q+1}$ $Pe \ll Re$

$\vec{\nabla}\cdot\vec{V}_p' = -\tau_p\vec{\nabla}((\vec{u}\cdot\vec{\nabla})\vec{u}) + o(\tau_p^2)$; $\vec{\nabla}\cdot((\vec{u}\cdot\vec{\nabla})\vec{u}) = \dots = -\frac{\Delta p'}{\overline{\rho}}$

$\langle\vec{V}_p'(\vec{\nabla}\cdot\vec{V}_p')\rangle = \dots = \frac{\tau_p}{\overline{\rho}}\langle\vec{u}\Delta p'\rangle + o(\tau_p^2)$; $p' = \frac{\overline{P}}{\overline{\rho}}\rho' + \frac{\overline{P}}{\overline{T}}\theta$

$$\langle\vec{u}\Delta p'\rangle = \lim_{\vec{x}\to\vec{y}}\langle u(t,\vec{x})\Delta p'(t,\vec{y})\rangle = -\int k^2\langle u(t,\vec{k})p'(t,-\vec{k})\rangle d\vec{k} = -\frac{\overline{P}}{\overline{T}}\int k^2\langle\vec{u}(t,\vec{k})\theta(t,-\vec{k})\rangle d\vec{k}$$

$\left|\frac{1}{\overline{\rho}}\langle\vec{u}\rho'\rangle\right| \ll \left|\frac{1}{\overline{T}}\langle\vec{u}\theta\rangle\right|$ for $Ma \ll 1$

$\langle u_i(\vec{k})\theta(-\vec{k})\rangle = -\tau(k)\langle u_i(\vec{k})u_j(-\vec{k})\rangle\nabla_j\overline{T}$; $\vec{V}^{eff} = -\frac{\tau\tau_p}{\overline{\rho}}\langle\vec{u}\Delta p'\rangle = -\frac{\overline{P}}{\overline{T}}\frac{\tau_p}{\overline{\rho}}\int k^2\tau(k)^2\langle u_i(\vec{k})u_j(-\vec{k})\rangle d\vec{k}(\nabla_j\overline{T})$

$$\int_{k_0}^{k_\nu}\tau^2(k)E(k)k^2dk = 4\tau_0\frac{2}{3}l_0^2\int\left(\frac{k}{k_0}\right)^{-\frac{4}{3}-\frac{5}{3}+2}d\left(\frac{k}{k_0}\right) = \frac{8}{3}\left(\frac{\tau_0}{l_0}\right)^2\int_1^{Re^{3/4}}x^{-1}dx = \frac{8}{3}\left(\frac{\tau_0}{l_0}\right)^2\ln x\Big|_1^{Re^{3/4}} = \frac{8}{3}\left(\frac{\tau_0}{l_0}\right)^2\cdot\frac{3}{4}\ln(Re) = 2\left(\frac{\tau_0}{l_0}\right)^2\ln(Re)$$

$\vec{V}^{eff} = -D_T\frac{\vec{\nabla}\overline{T}}{\overline{T}} - \frac{2}{3\gamma}c_s^2\tau_p\ln(Re)\frac{\vec{\nabla}\overline{T}}{\overline{T}}$; $\gamma = \frac{c_p}{c_v}$, $c_s^2 = \gamma\frac{\overline{P}}{\overline{\rho}}$

$$\frac{\partial\overline{n}}{\partial t} + \vec{\nabla}\left[\overline{n}\left(-\frac{\tau_p}{3}\vec{\nabla}\langle u^2\rangle + \tau_p\vec{g}\right) + \overline{n}\left(-D_T\frac{\vec{\nabla}\overline{T}}{\overline{T}} - \frac{2}{3\gamma}c_s^2\tau_p\ln Re\frac{\vec{\nabla}\overline{T}}{\overline{T}}\right) - (D_T + D^{(n)})\vec{\nabla}\overline{n}\right] = 0$$

$$\frac{\overline{n}}{n_0} = \left(\frac{\overline{T}(z)}{T_0}\right)^{-\frac{\alpha D_T}{D_T + D^{(n)}}}\exp\left[\int\frac{\frac{\tau_p}{3}\vec{\nabla}\langle u^2\rangle - \vec{g}\tau_p}{D_T + D^{(n)}}dz\right]$$

$\overline{n}(z = z_0) = n_0$, $\overline{T}(z = z_0) = T_0$, $F(z = z_0) = 0$